

Review Questions

All Topics, Separated by Paper and Section

Complete each section by the required date. You need to find Ms. Griffith to show her you're done.

<i>Date Due (total time)</i>	<i>Questions (Time Allotted)</i>	<i>Time Taken to Complete</i>	<i>Peer Signature (ONLY IF COMPLETE)</i>
<i>Mon. 4/22, by 12PM (89 min over 3 days)</i>	<i>Paper 1: #1-7 (44 min) Paper 2: #47-51 (45 min)</i>		
<i>Tues. 4/23, by 12PM (69 min over 1 day)</i>	<i>Paper 1: #8-13 (36 min) Paper 2: #52-54 (33 min)</i>		
<i>Wed. 4/24, by 12PM (68 min over 1 day)</i>	<i>Paper 1: #14-19 (36 min) Paper 2: #55-57 (32 min)</i>		
<i>Thurs. 4/25, by 12PM (93 min over 1 day)</i>	<i>Paper 1: #20-25 (36 min) Paper 2: #58-60 (57 min)</i>		
<i>Fri. 4/26, by 12PM (94 min over 1 day)</i>	<i>Paper 1: #26-33 (46 min) Paper 2: #61-63 (48 min)</i>		
<i>Tues. 4/30, by 12PM (122 min over 4 days)</i>	<i>Paper 1: #34-44 (66 min) Paper 2: #64-67 (56 min)</i>		
<i>Wed. 5/1, by 12PM (26 min over 1 day)</i>	<i>Paper 1: #45-46 (12 min) Paper 2: #68 (14 min)</i>		

Paper 1

Answer these questions on the boxes or in the space given.

1. Consider the numbers $\sqrt{3}$, 6, $2\frac{1}{2}$, π , -5 , and the sets \mathbb{N} , \mathbb{Z} , and \mathbb{Q} . Complete the following table by placing a tick in the appropriate box if the number is an element of the set.

	$\sqrt{3}$	6	$2\frac{1}{2}$	π	-5
\mathbb{N}					
\mathbb{Z}					
\mathbb{Q}					

(Total 6 marks)

2. Given $p = x - \frac{\sqrt{y}}{z}$, $x = 1.775$, $y = 1.44$ and $z = 48$,

(a) calculate the value of p .

(2)

Barry **first** writes x , y and z correct to one significant figure and **then** uses these values to estimate the value of p .

(b) (i) Write down x , y and z each correct to one significant figure.

(ii) Write down Barry's estimate of the value of p .

(2)

(c) Calculate the percentage error in Barry's estimate of the value of p .

(2)

(Total 6 marks)

3. (a) Given $x = 2.6 \times 10^4$ and $y = 5.0 \times 10^{-8}$, calculate the value of $w = x \times y$. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Which **two** of the following statements about the nature of x , y and w above are **incorrect**?

(i) $x \in \mathbb{N}$

(v) $x + y \in \mathbb{R}$

(ii) $y \in \mathbb{Z}$

(vi) $\frac{1}{w} < x$

(iii) $y \in \mathbb{Q}$

(iv) $w < y$

(Total 8 marks)

4. A field is 91.4 m long and 68.5 m wide.

(a) Calculate the area of the field in m^2 .

(b) Calculate the area of the field in cm^2 .

(c) Express your answer to (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(Total 6 marks)

5. The planet Earth takes one year to revolve around the Sun. Assume that a year is 365 days and the path of the Earth around the Sun is the circumference of a circle of radius 150 000 000 km.

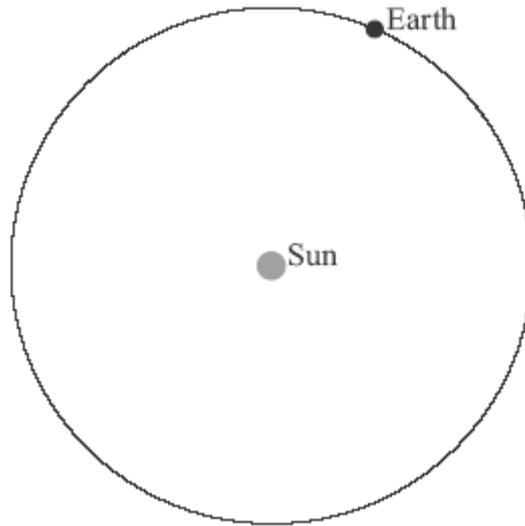


diagram not to scale

- (a) Calculate the distance travelled by the Earth in **one day**. (4)
- (b) Give your answer to part (a) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. (2)
- (Total 6 marks)**

6. A teacher earns an annual salary of 45 000 USD for the first year of her employment. Her annual salary increases by 1750 USD each year.

- (a) Calculate the annual salary for the fifth year of her employment. (3)

She remains in this employment for 10 years.

- (b) Calculate the **total** salary she earns in this employment during these 10 years. (3)

(Total 6 marks)

7. The first term of an arithmetic sequence is 3 and the sum of the first two terms is 11.

- (a) Write down the second term of this sequence. (1)
- (b) Write down the common difference of this sequence. (1)
- (c) Write down the fourth term of this sequence. (1)
- (d) The n^{th} term is the first term in this sequence greater than 1000. Find the value of n . (3)

(Total 6 marks)

8. The seventh term, u_7 , of a geometric sequence is 108. The eighth term, u_8 , of the sequence is 36.

(a) Write down the common ratio of the sequence.

(1)

(b) Find u_1 .

(2)

The sum of the first k terms in the sequence is 118 096.

(c) Find the value of k .

(3)

(Total 6 marks)

9. The annual fees paid to a school for the school years 2000, 2001 and 2002 increase as a geometric progression. The table below shows the fee structure.

Year	Fees (USD)
2000	8000.00
2001	8320.00
2002	8652.80

(a) Calculate the common ratio for the increasing sequence of fees.

(2)

In parts (b) and (c) give your answer correct to 2 decimal places.

The fees continue to increase in the same ratio.

(b) Find the fees paid for 2006.

(2)

A student attends the school for eight years, starting in 2000.

(c) Find the **total** fees paid for these eight years.

(2)

(Total 6 marks)

10. The number of cells, C , in a culture is given by the equation $C = p \times 2^{0.5t} + q$, where t is the time in hours measured from 12:00 on Monday and p and q are constants.

The number of cells in the culture at 12:00 on Monday is 47.

The number of cells in the culture at 16:00 on Monday is 53.

Use the above information to

(a) write down two equations in p and q ;

(2)

(b) calculate the value of p and of q ;

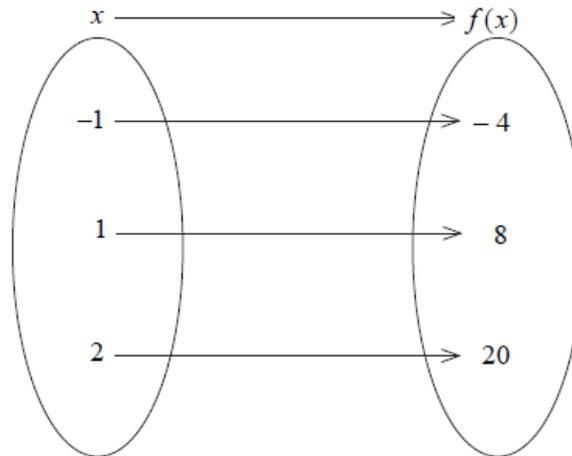
(2)

(c) find the number of cells in the culture at 22:00 on Monday.

(2)

(Total 6 marks)

11. A quadratic function, $f(x) = ax^2 + bx$, is represented by the mapping diagram below.



- (a) Use the mapping diagram to write down **two** equations in terms of a and b . (2)
- (b) Find the value of
- (i) a ;
- (ii) b . (2)
- (c) Calculate the x -coordinate of the vertex of the graph of $f(x)$. (2)

(Total 6 marks)

12. The length of a square garden is $(x + 1)$ m. In one of the corners a square of 1 m length is used only for grass. The rest of the garden is only for planting roses and is shaded in the diagram below.

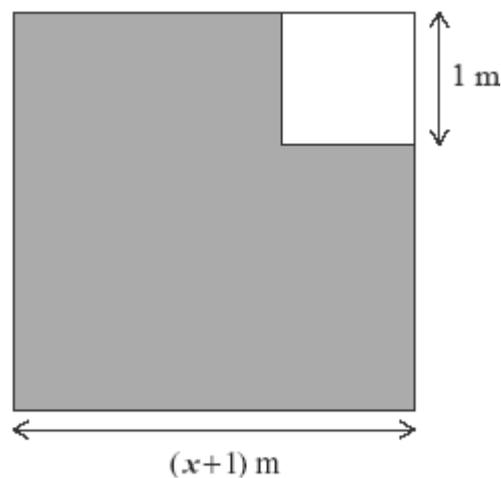


diagram not to scale

The area of the shaded region is A .

- (a) Write down an expression for A in terms of x . (1)
- (b) Find the value of x given that $A = 109.25 \text{ m}^2$. (3)
- (c) The owner of the garden puts a fence around the shaded region. Find the length of this fence. (2)

(Total 6 marks)

13. In the diagram, $\hat{BAC} = 90^\circ$. The length of the three sides are x cm, $(x + 7)$ cm and $(x + 8)$ cm.

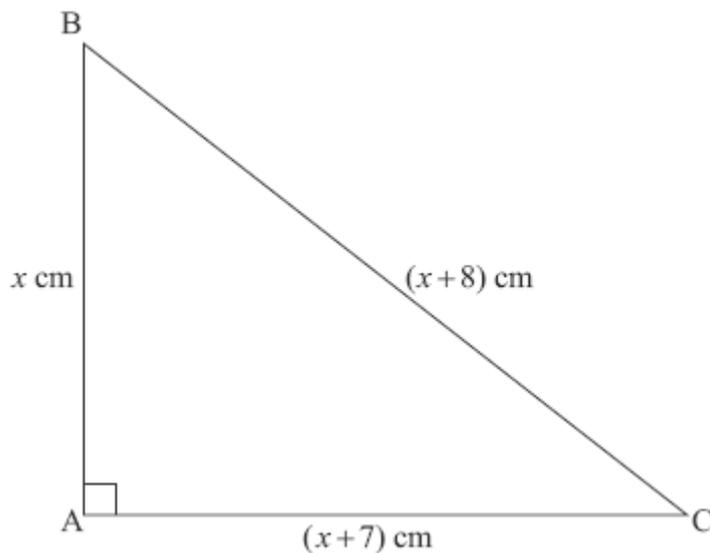


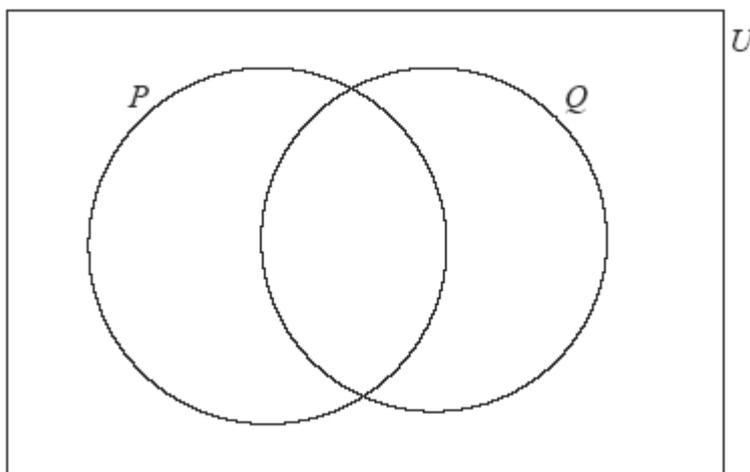
diagram not to scale

- (a) Write down and **simplify** a quadratic equation in x that links the three sides of the triangle. (3)
- (b) Solve the quadratic equation found in part (a). (2)
- (c) Write down the value of the perimeter of the triangle. (1)

(Total 6 marks)

14. The sets P , Q and U are defined as

$U = \{\text{Real Numbers}\}$, $P = \{\text{Positive Numbers}\}$ and $Q = \{\text{Rational Numbers}\}$.

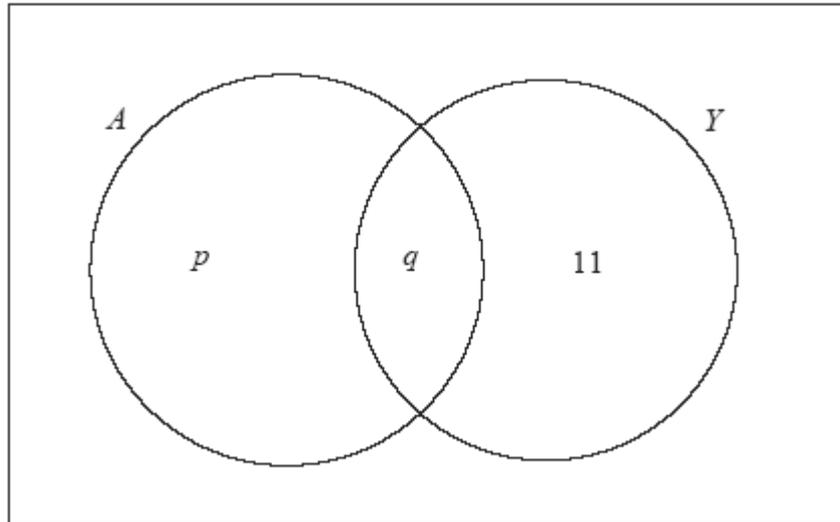


Write down in the correct region on the Venn diagram the numbers

$\frac{22}{7}$, 5×10^{-2} , $\sin(60^\circ)$, 0 , $\sqrt[3]{-8}$, $-\pi$

(Total 6 marks)

15. A fitness club has 60 members. 35 of the members attend the club's aerobics course (A) and 28 members attend the club's yoga course (Y). 17 members attend both courses. A Venn diagram is used to illustrate this situation.



- (a) Write down the value of q . (1)
- (b) Find the value of p . (2)
- (c) Calculate the number of members of the fitness club who attend neither the aerobics course (A) nor the yoga course (Y). (2)
- (d) Shade, on your Venn diagram, $A' \cap Y$. (1)
- (Total 6 marks)**

16. In a particular school, students must choose at least one of three optional subjects: art, psychology or history.

Consider the following propositions

a : I choose art,
 p : I choose psychology,
 h : I choose history.

- (a) Write, in words, the compound proposition
 $\neg h \Rightarrow (p \vee a)$. (3)
- (b) Complete the truth table for $\neg a \Rightarrow p$.

a	p	$\neg a$	$\neg a \Rightarrow p$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

- (c) State whether $\neg a \Rightarrow p$ is a tautology, a contradiction or neither. Justify your answer. (2)
- (Total 6 marks)**

17. (a) Complete the following truth table.

p	q	$p \Rightarrow \neg q$
T	T	F
T	F	T
F	T	F
F	F	T

(2)

Consider the propositions

p : Cristina understands logic

q : Cristina will do well on the logic test.

- (b) Write down the following compound proposition in symbolic form.

“If Cristina understands logic then she will do well on the logic test”

(2)

- (c) Write down in words the contrapositive of the proposition given in part (b).

(2)

(Total 6 marks)

18. (a) List the elements of the set $A = \{x \mid -4 \leq x \leq 2, x \text{ is an integer}\}$.

(1)

A number is chosen at random from set A .

Write down the probability that the number chosen is

- (b) a negative integer;

(2)

- (c) a positive even integer;

(1)

- (d) an odd integer less than -1 .

(2)

(Total 6 marks)

19. A survey was carried out at an international airport. A number of travellers were interviewed and asked for their flight destinations. The results are shown in the table below.

Destination	America	Africa	Asia
Number of males	45	62	37
Number of females	35	46	25

One traveller is to be chosen at random from all those interviewed.

- (a) Find the probability that this traveller was going to Africa.

(2)

One female traveller is to be chosen at random from all those interviewed.

- (b) Find the probability that this female traveller was going to Asia.

(2)

One traveller is to be chosen at random from those **not** going to America.

- (c) Find the probability that the chosen traveller is female.

(2)

(Total 6 marks)

20. In a research project on the relation between the gender of 150 science students at college and their degree subject, the following set of data is collected.

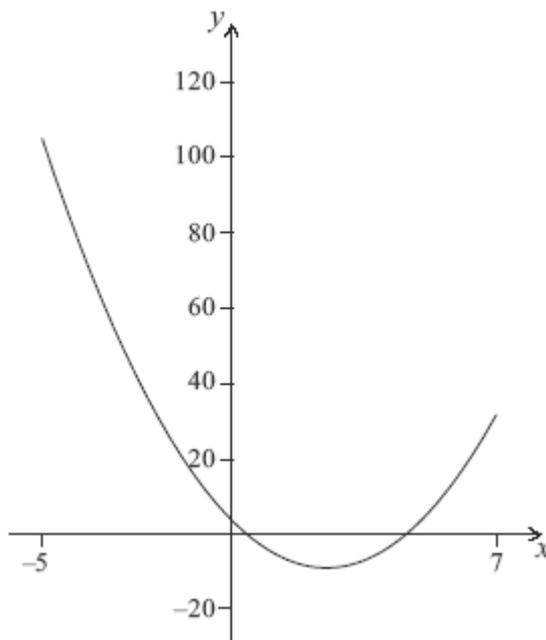
		Degree Subject		
		Biology	Physics	Chemistry
Gender	Male	40	16	35
	Female	15	24	20

Find the probability that a student chosen at random

- (a) is male; (2)
- (b) is either male or studies Chemistry; (2)
- (c) studies Physics, given that the student is male. (2)

(Total 6 marks)

21. The graph of $y = 2x^2 - rx + q$ is shown for $-5 \leq x \leq 7$.



The graph cuts the y -axis at $(0, 4)$.

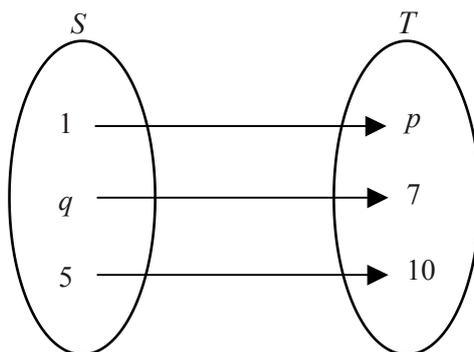
- (a) Write down the value of q . (1)

The axis of symmetry is $x = 2.5$.

- (b) Find the value of r . (2)
- (c) Write down the minimum value of y . (1)
- (d) Write down the range of y . (2)

(Total 6 marks)

22. (a) $f: x \rightarrow 3x - 5$ is a mapping from the set S to the set T as shown below.



Find the values of p and q . (2)

- (b) A function g is such that $g(x) = \frac{3}{(x-2)^2}$.

(i) State the domain of the function $g(x)$. (2)

(ii) State the range of the function $g(x)$. (1)

(iii) Write down the equation of the vertical asymptote. (1)

(Total 6 marks)

23. A plumber in Australia charges 90 AUD per hour for work, plus a fixed cost. His total charge is represented by the cost function $C = 60 + 90t$, where t is in hours.

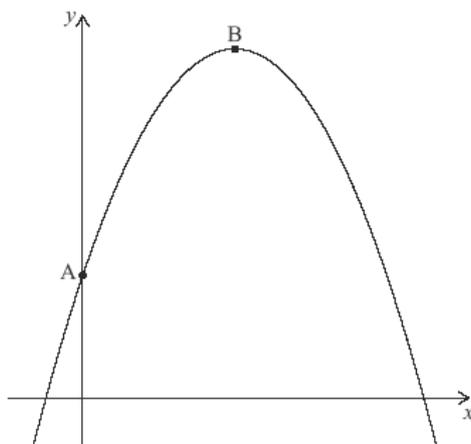
(a) Write down the fixed cost. (1)

(b) It takes $3\frac{1}{2}$ hours to complete a job for Paula. Find the total cost. (2)

(c) Steve received a bill for 510 AUD. Calculate the time it took the plumber to complete the job. (3)

(Total 6 marks)

24. The graph of the quadratic function $f(x) = 3 + 4x - x^2$ intersects the y -axis at point A and has its vertex at point B.



(a) Find the coordinates of B. (3)

Another point, C, which lies on the graph of $y = f(x)$ has the same y -coordinate as A.

(b) (i) Plot and label C on the graph above.

(ii) Find the x -coordinate of C. (3)

(Total 6 marks)

25. Shiyun bought a car in 1999. The value of the car V , in USD, is depreciating according to the exponential model

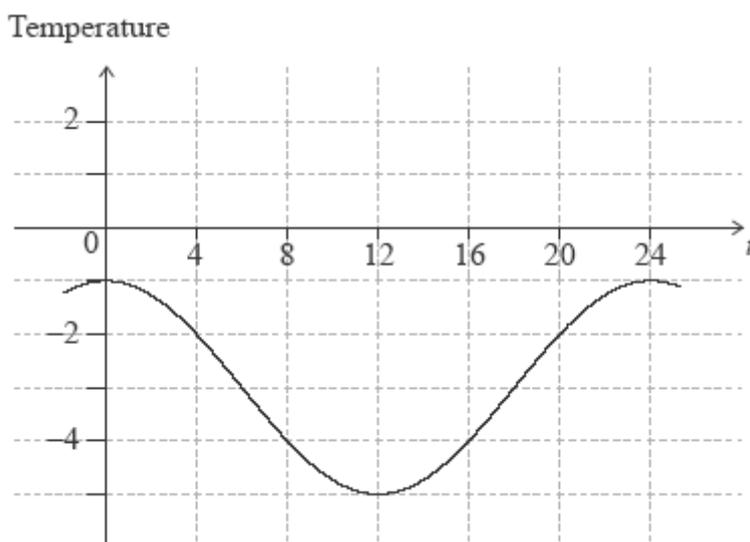
$$V = 25\,000 \times 1.5^{-0.2t}, \quad t \geq 0,$$

where t is the time, in years, that Shiyun has owned the car.

- (a) Write down the value of the car when Shiyun bought it. (1)
- (b) Calculate the value of the car three years after Shiyun bought it. Give your answer correct to **two decimal places**. (2)
- (c) Calculate the time for the car to depreciate to half of its value since Shiyun bought it. (3)

(Total 6 marks)

26. The temperature in degrees Celsius during a 24 hour period is shown on the graph and is given by the function $f(t) = a \cos(bt) + c$, where a , b and c are constants, t is the time in hours and (bt) is measured in degrees.



- (a) Write down the value of a . (1)
- (b) Find the value of b . (2)
- (c) Write down the value of c . (1)
- (d) Write down the interval of time during which the temperature is increasing from $-4\text{ }^{\circ}\text{C}$ to $-2\text{ }^{\circ}\text{C}$. (2)

(Total 6 marks)

27. The function $f(x) = 5 - 3(2^{-x})$ is defined for $x \geq 0$.

- (a) (i) On the axes below sketch the graph of $f(x)$ and show the behaviour of the curve as x increases.
 (ii) Write down the coordinates of any intercepts with the axes.



- (b) Draw the line $y = 5$ on your sketch.
 (c) Write down the number of solutions to the equation $f(x) = 5$.

(4)

(1)

(1)

(Total 6 marks)

28. The straight line L passes through the points $A(-1, 4)$ and $B(5, 8)$.

- (a) Calculate the gradient of L .
 (b) Find the equation of L .

(2)

(2)

The line L also passes through the point $P(8, y)$.

- (c) Find the value of y .

(2)

(Total 6 marks)

29. A room is in the shape of a cuboid. Its floor measures 7.2 m by 9.6 m and its height is 3.5 m.

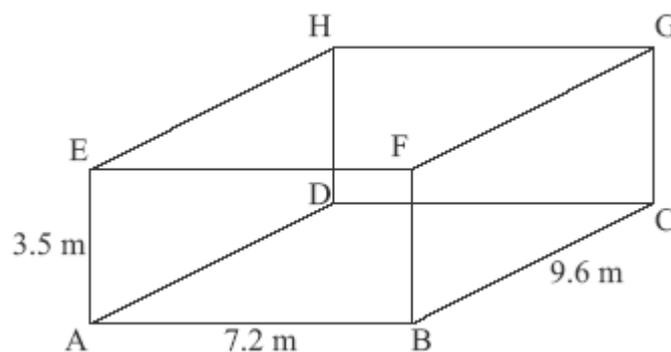


diagram not to scale

- (a) Calculate the length of AC .
 (b) Calculate the length of AG .
 (c) Calculate the angle that AG makes with the floor.

(2)

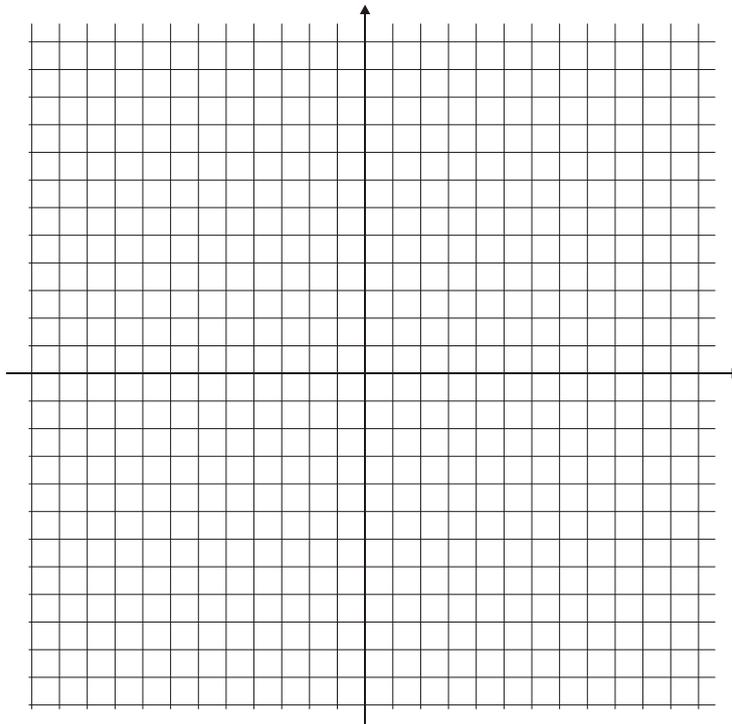
(2)

(2)

(Total 6 marks)

30. Three points A (1, 3), B (4, 10) and C (7, -1) are joined to form a triangle. The mid-point of AB is D and the mid-point of AC is E.

(a) Plot the points A, B, C, on the grid.



(b) Find the distance DE.

(Total 6 marks)

31. In the diagram, AD = 4 m, AB = 9 m, BC = 10 m, $\hat{BDA} = 90^\circ$ and $\hat{DBC} = 100^\circ$.

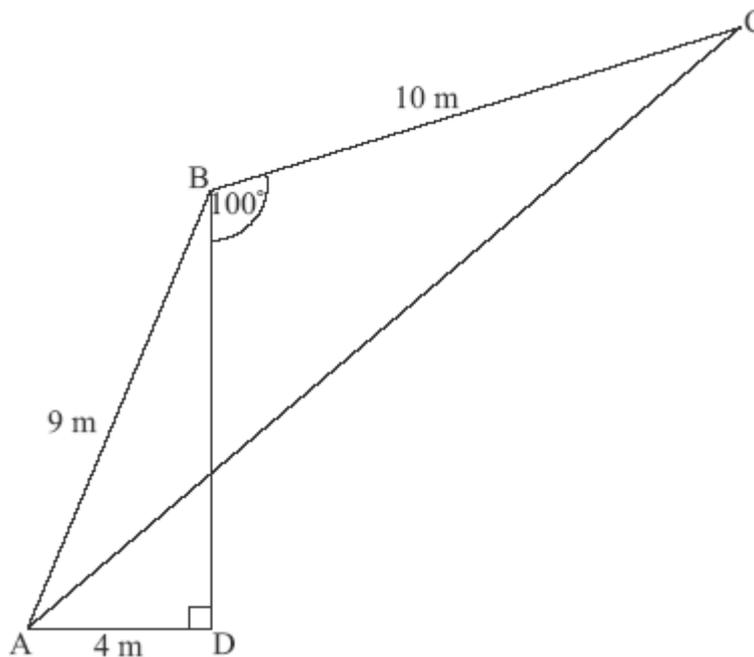


diagram not to scale

(a) Calculate the size of \hat{ABC} .

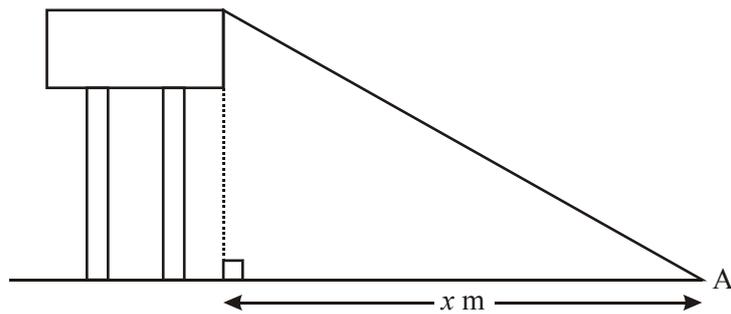
(3)

(b) Calculate the length of AC.

(3)

(Total 6 marks)

32. The diagram shows a water tower standing on horizontal ground. The height of the tower is 26.5 m.



From a point A on the ground the angle of elevation to the top of the tower is 28° .

- (a) On the diagram, show and label the angle of elevation, 28° .
 (b) Calculate, **correct to the nearest metre**, the distance x m.

(Total 4 marks)

33. Triangle ABC is such that AC is 7 cm, angle $\hat{A}BC$ is 65° and angle $\hat{A}CB$ is 30° .

- (a) Sketch the triangle writing in the side length and angles.
 (b) Calculate the length of AB.
 (c) Find the area of triangle ABC.

(1)

(2)

(3)

(Total 6 marks)

34. The table below shows the frequency distribution of the number of dental fillings for a group of 25 children.

Number of fillings	0	1	2	3	4	5
Frequency	4	3	8	q	4	1

- (a) Find the value of q .
 (b) Use your graphic display calculator to find
 (i) the mean number of fillings;
 (ii) the median number of fillings;
 (iii) the standard deviation of the number of fillings.

(2)

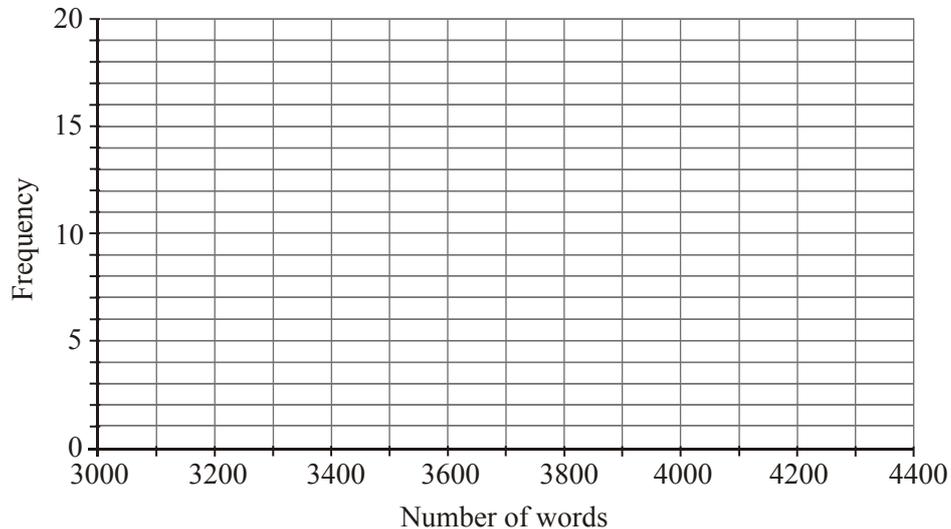
(4)

(Total 6 marks)

35. The table below shows the number of words in the extended essays of an IB class.

Number of words	$3200 \leq w < 3400$	$3400 \leq w < 3600$	$3600 \leq w < 3800$	$3800 \leq w < 4000$	$4000 \leq w < 4200$
Frequency	2	5	8	17	3

(a) Draw a histogram on the grid below for the data in this table.



(3)

(b) Write down the modal group.

(1)

The maximum word count is 4000 words.

(c) Write down the probability that a student chosen at random is on or over the word count.

(2)

(Total 6 marks)

36. (a) Complete the following table of values for the height and weight of seven students.

	Values	Mode	Median	Mean	Standard deviation
Height (cm)	151, 158, 171, 163, 184, 148, 171			164	11.7
Weight (kg)	53, 61, 58, 82, 45, 72, 82	82	61		

(4)

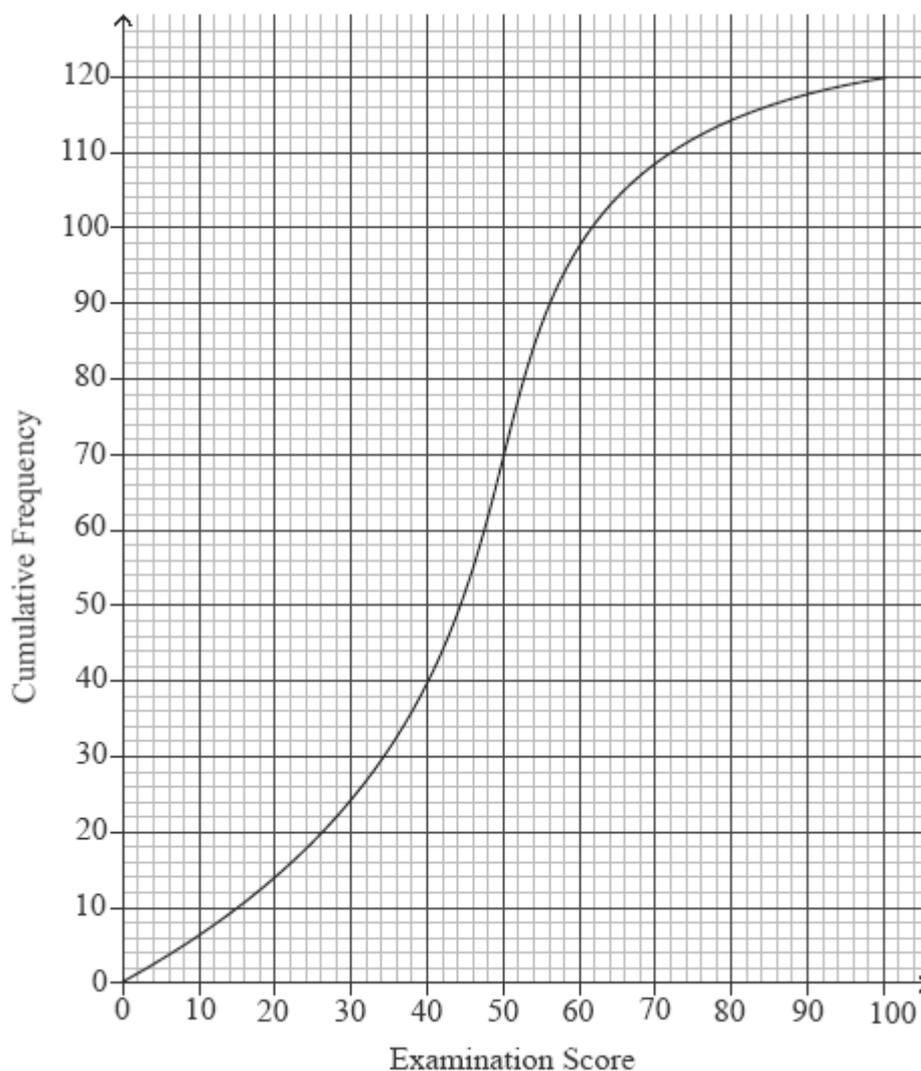
The ages (in months) of seven students are 194, 205, 208, 210, 200, 226, 223.

(b) Represent these values in an ordered stem and leaf diagram.

(2)

(Total 6 marks)

37. 120 Mathematics students in a school sat an examination. Their scores (given as a percentage) were summarized on a cumulative frequency diagram. This diagram is given below.



- (a) Complete the grouped frequency table for the students.

Examination Score x (%)	$0 \leq x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	14	26			

(3)

- (b) Write down the mid-interval value of the $40 < x \leq 60$ interval.

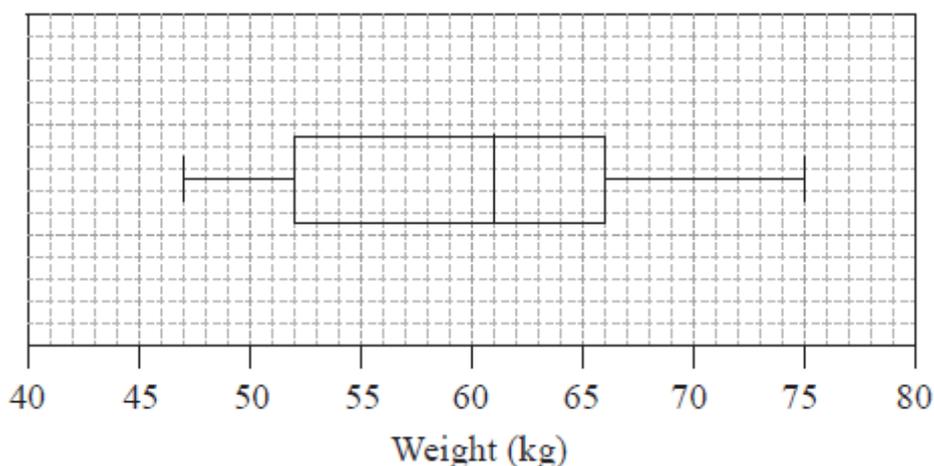
(1)

- (c) Calculate an estimate of the mean examination score of the students.

(2)

(Total 6 marks)

38. The weights in kg, of 80 adult males, were collected and are summarized in the box and whisker plot shown below.



- (a) Write down the median weight of the males. (1)
- (b) Calculate the interquartile range. (2)
- (c) Estimate the number of males who weigh between 61 kg and 66 kg. (1)
- (d) Estimate the mean weight of the lightest 40 males. (2)

(Total 6 marks)

39. The marks obtained by 8 candidates in Physics and Chemistry tests are given below.

Physics (x)	6	8	10	11	10	5	4	12
Chemistry (y)	8	11	14	13	11	7	5	15

- (a) Write down the product moment correlation coefficient, r . (1)
- (b) Write down, in the form $y = mx + c$, the equation of the regression line y on x for the 8 candidates. (2)

A ninth candidate obtained a score of 7 in the Physics test but was absent for the Chemistry test.

- (c) Use your answer to (b) to estimate the score he would have obtained on the Chemistry test. (2)
- (d) Give a reason why it is valid to use this regression line to estimate the score on the Chemistry test. (1)

(Total 6 marks)

40. A shop keeper recorded daily sales s of ice cream along with the daily maximum temperature t °C. The results for one week are shown below.

t	29	31	34	23	19	20	27
s	104	92	112	48	56	72	66

- (a) Write down the equation of the regression line for s on t . (3)
- (b) Use your equation to predict the ice cream sales on a day when the maximum temperature is 24 °C. Give your answer correct to the nearest whole number. (3)

(Total 6 marks)

41. Tony wants to carry out a χ^2 test to determine whether or not a person's choice of one of the three professions – engineering, medicine or law – is influenced by the person's sex (gender).
- (a) State the null hypothesis, H_0 , for this test. (1)
- (b) Write down the number of degrees of freedom. (1)

Of the 400 people Tony interviewed, 220 were male and 180 were female.
80 of the people had chosen engineering as a profession.

- (c) Calculate the expected number of female engineers. (2)

Tony used a 5 % level of significance for his test and obtained a p -value of 0.0634 correct to 3 significant figures.

- (d) State Tony's conclusion to the test. Give a reason for this conclusion. (2)
- (Total 6 marks)**

42. Consider the function $f(x) = 2x^3 - 5x^2 + 3x + 1$.

- (a) Find $f'(x)$. (3)
- (b) Write down the value of $f'(2)$. (1)
- (c) Find the equation of the tangent to the curve of $y = f(x)$ at the point (2, 3). (2)
- (Total 6 marks)**

43. (a) Differentiate the function $y = x^2 + 3x - 2$.

- (b) At a certain point (x, y) on this curve the gradient is 5. Find the co-ordinates of this point. (4)
- (Total 6 marks)**

44. The exchange rates between the British pound (GBP) and the United States dollar (USD) and between the USD and the euro (EUR) are given below.

1 GBP	2.034 USD
1 USD	0.632 EUR

- (a) Find the exchange rate between GBP and EUR in the form $1 \text{ GBP} = k \text{ EUR}$, where k is a constant. Give your answer correct to **two decimal places**. (2)

Isabella changes 400 USD into euros and is charged 2 % commission.

- (b) Calculate how many euros she receives. Give your answer correct to **two decimal places**. (4)
- (Total 6 marks)**

45. Astrid invests 1200 euros for five years at a nominal annual interest rate of 7.2 %, **compounded monthly**.

- (a) Find the interest Astrid has earned during the five years of her investment. **Give your answer correct to two decimal places**. (3)

Helen invests 1200 euros in an annual **simple interest** scheme for five years. She earns **the same** interest as Astrid.

- (b) Find the simple interest rate of this scheme. (3)
- (Total 6 marks)**

46. The table below shows the **monthly** repayments per \$10 000 borrowed for various nominal annual interest rates.

Loan term (years)	Table of monthly repayments in \$, per \$10 000		
	Annual interest rate		
	7%	8%	9%
5	198.0112	202.7634	207.5836
10	116.1085	121.3276	126.6758
15	89.8828	95.5652	101.4267
20	77.5299	83.6440	89.9726
25	70.6779	77.1816	83.9196

Beryl borrows \$150 000 to buy an apartment at an interest rate of 8%, to be repaid over 20 years.

- (a) Calculate Beryl's exact monthly repayment.
- (b) Find the exact amount of **interest** paid for the loan over the 20 years.

(Total 6 marks)

Paper 2

Answer these questions on appropriate separate paper. ONLY graphs go on graph paper.

47. Jenny has a circular cylinder with a lid. The cylinder has height 39 cm and diameter 65 mm.
- (a) Calculate the volume of the cylinder in cm^3 . Give your answer correct to two decimal places. (3)
- The cylinder is used for storing tennis balls.
Each ball has a radius of 3.25 cm.
- (b) Calculate how many balls Jenny can fit in the cylinder if it is filled to the top. (1)
- (c) (i) Jenny fills the cylinder with the number of balls found in part (b) and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls.
(ii) Convert your answer to (c) (i) into cubic metres. (4)
- (Total 8 marks)

48. Consider the arithmetic sequence 1, 4, 7, 10, 13, ...
- (a) Find the value of the eleventh term. (2)
- (b) The sum of the first n terms of this sequence is $\frac{n}{2}(3n - 1)$.
- (i) Find the sum of the first 100 terms in this arithmetic sequence.
(ii) The sum of the first n terms is 477.
- (a) Show that $3n^2 - n - 954 = 0$.
(b) Using your graphic display calculator or otherwise, find the number of terms, n . (6)
- (Total 8 marks)

49. An arithmetic sequence is defined as
- $$u_n = 135 + 7n, \quad n = 1, 2, 3, \dots$$
- (a) Calculate u_1 , the first term in the sequence. (2)
- (b) Show that the common difference is 7. (2)
- S_n is the sum of the first n terms of the sequence.
- (c) Find an expression for S_n . Give your answer in the form $S_n = An^2 + Bn$, where A and B are constants. (3)
- The first term, v_1 , of a geometric sequence is 20 and its fourth term v_4 is 67.5.
- (d) Show that the common ratio, r , of the geometric sequence is 1.5. (2)
- T_n is the sum of the first n terms of the geometric sequence.
- (e) Calculate T_7 , the sum of the first seven terms of the geometric sequence. (2)
- (f) Use your graphic display calculator to find the smallest value of n for which $T_n > S_n$. (2)
- (Total 13 marks)

50. The curve $y = px^2 + qx - 4$ passes through the point $(2, -10)$.
- (a) Use the above information to write down an equation in p and q . (2)

The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1.

- (b) (i) Find $\frac{dy}{dx}$.
- (ii) Hence, find a second equation in p and q . (3)
- (c) Solve the equations to find the value of p and of q . (3)

(Total 8 marks)

51. (a) Factorize $3x^2 + 13x - 10$. (2)

- (b) Solve the equation $3x^2 + 13x - 10 = 0$. (2)

Consider a function $f(x) = 3x^2 + 13x - 10$.

- (c) Find the equation of the axis of symmetry on the graph of this function. (2)
- (d) Calculate the minimum value of this function. (2)

(Total 8 marks)

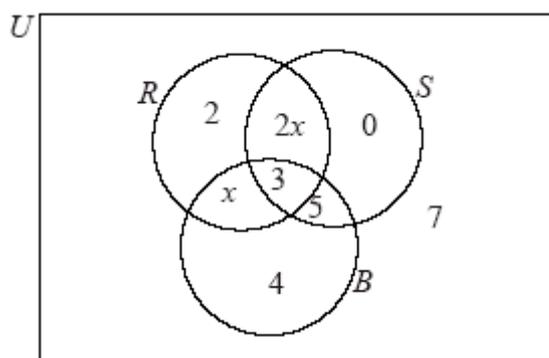
52. 100 students are asked what they had for breakfast on a particular morning. There were three choices: cereal (X), bread (Y) and fruit (Z). It is found that

10 students had all three
 17 students had bread and fruit only
 15 students had cereal and fruit only
 12 students had cereal and bread only
 13 students had only bread
 8 students had only cereal
 9 students had only fruit

- (a) Represent this information on a Venn diagram. (4)
- (b) Find the number of students who had none of the three choices for breakfast. (2)
- (c) Write down the percentage of students who had fruit for breakfast. (2)
- (d) Describe in words what the students in the set $X \cap Y'$ had for breakfast. (2)
- (e) Find the probability that a student had **at least** two of the three choices for breakfast. (2)
- (f) Two students are chosen at random. Find the probability that both students had all three choices for breakfast. (3)

(Total 15 marks)

53. A survey was carried out in a year 12 class. The pupils were asked which pop groups they like out of the *Rockers* (R), the *Salseros* (S), and the *Bluers* (B). The results are shown in the following diagram.



- (a) Write down $n(R \cap S \cap B)$. (1)
- (b) Find $n(R')$. (2)
- (c) Describe which groups the pupils in the set $S \cap B$ like. (2)
- (d) Use set notation to describe the group of pupils who like the *Rockers* and the *Bluers* but do not like the *Salseros*. (2)

There are 33 pupils in the class.

- (e) (i) Find x . (3)
- (ii) Find the number of pupils who like the *Rockers*. (3)

(Total 10 marks)

54. Three propositions are given as

p : It is snowing q : The roads are open r : We will go skiing

- (a) Write the following compound statement in symbolic form. (2)
- “It is snowing and the roads are not open.”
- (b) Write the following compound statement in words. (3)

$$(\neg p \wedge q) \Rightarrow r$$

An incomplete truth table for the compound proposition $(\neg p \wedge q) \Rightarrow r$ is given below.

- (c) Copy and complete the truth table **on your answer paper**.

p	q	r	$\neg p$	$\neg p \wedge q$	$(\neg p \wedge q) \Rightarrow r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

(3)

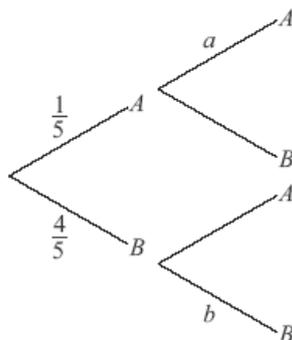
(Total 8 marks)

55. Let p stand for the proposition “I will walk to school”. Let q stand for the proposition “the sun is shining”.

- (a) Write the following statements in symbolic logic form
- (i) “If the sun is shining then I will walk to school.”
- (ii) “If I do not walk to school then the sun is not shining.” (4)
- (b) Write down, in words, the converse of the statement
 “If the sun is shining then I will walk to school.” (2)

(Total 6 marks)

56. (a) Phoebe chooses a biscuit from a blue tin on a shelf. The tin contains one chocolate biscuit and four plain biscuits. She eats the biscuit and chooses another one from the tin. The tree diagram below represents the situation with the four possible outcomes where A stands for chocolate biscuit and B for plain biscuit.

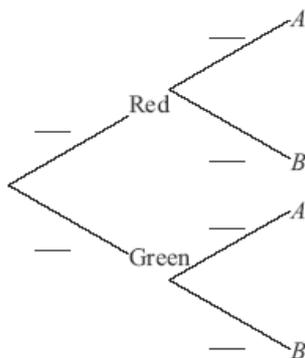


- (i) Write down the value of a .
- (ii) Write down the value of b .
- (iii) Find the probability that both biscuits are plain.

(6)

On another shelf there are two tins, one red and one green. The red tin contains three chocolate biscuits and seven plain biscuits and the green tin contains one chocolate biscuit and four plain biscuits. Andrew randomly chooses either the red or the green tin and randomly selects a biscuit.

(b) **Copy and complete** the tree diagram below.

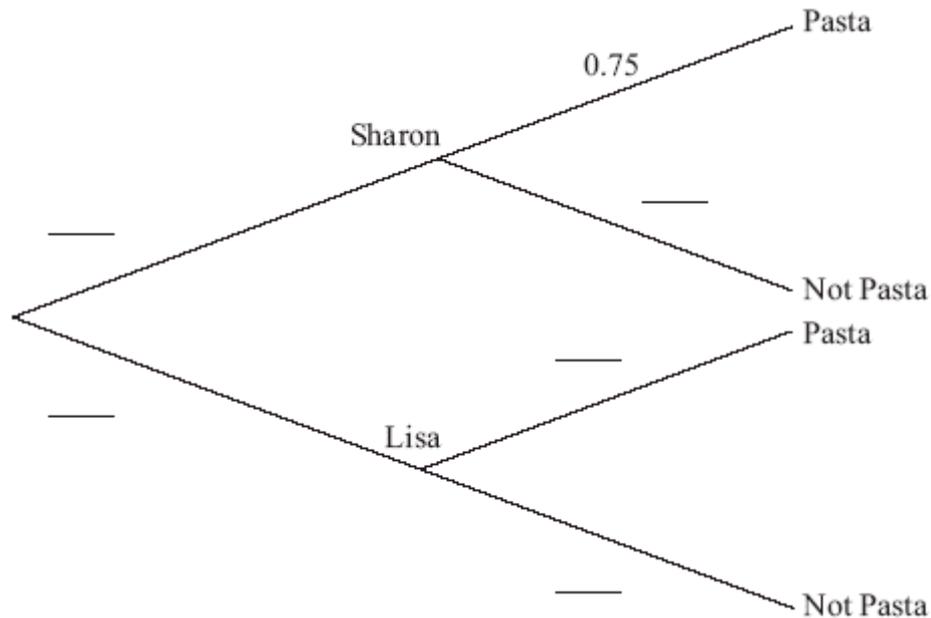


(3)

- (c) Find the probability that
- (i) he chooses a chocolate biscuit;
- (ii) he chooses a biscuit from the red tin given that it is a chocolate biscuit. (6)

(Total 15 marks)

57. Sharon and Lisa share a flat. Sharon cooks dinner three nights out of ten. If Sharon does not cook dinner, then Lisa does. If Sharon cooks dinner the probability that they have pasta is 0.75. If Lisa cooks dinner the probability that they have pasta is 0.12.
- (a) **Copy and complete** the tree diagram to represent this information.



- (b) Find the probability that Lisa cooks dinner and they do not have pasta. (2)
- (c) Find the probability that they do not have pasta. (3)
- (d) Given that they do not have pasta, find the probability that Lisa cooked dinner. (3)
- (Total 11 marks)**

58. The function $f(x)$ is defined by $f(x) = 1.5x + 4 + \frac{6}{x}$, $x \neq 0$.

- (a) Write down the equation of the vertical asymptote. (2)
- (b) Find $f'(x)$. (3)
- (c) Find the gradient of the graph of the function at $x = -1$. (2)
- (d) Using your answer to part (c), decide whether the function $f(x)$ is increasing or decreasing at $x = -1$. Justify your answer. (2)
- (e) Sketch the graph of $f(x)$ for $-10 \leq x \leq 10$ and $-20 \leq y \leq 20$. (4)

P_1 is the local maximum point and P_2 is the local minimum point on the graph of $f(x)$.

- (f) Using your graphic display calculator, write down the coordinates of
- (i) P_1 ;
- (ii) P_2 . (4)
- (g) Using your sketch from (e), determine the range of the function $f(x)$ for $-10 \leq x \leq 10$. (3)

(Total 20 marks)

59. Consider the function $f(x) = x^3 - 3x^2 - 24x + 30$.
- (a) Write down $f(0)$. (1)
- (b) Find $f'(x)$. (3)
- (c) Find the gradient of the graph of $f(x)$ at the point where $x = 1$. (2)

The graph of $f(x)$ has a local maximum point, M, and a local minimum point, N.

- (d) (i) Use $f'(x)$ to find the x -coordinate of M and of N. (5)
- (ii) Hence or otherwise write down the coordinates of M and of N. (5)
- (e) Sketch the graph of $f(x)$ for $-5 \leq x \leq 7$ and $-60 \leq y \leq 60$. Mark clearly M and N on your graph. (4)

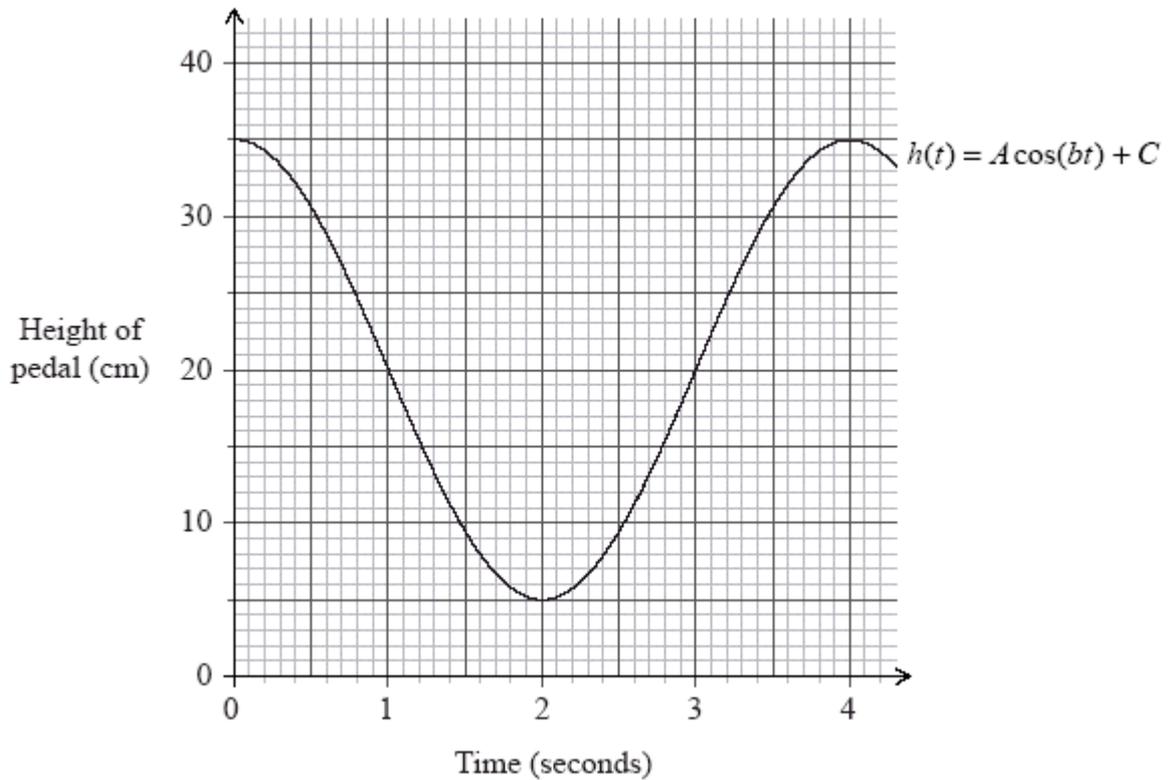
Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$.

- (f) (i) Find the x -coordinate of A and of B. (6)
- (ii) Find the y -coordinate of B. (6)
- (Total 21 marks)**

60. (a) Sketch the graph of $y = 2^x$ for $-2 \leq x \leq 3$. Indicate clearly where the curve intersects the y -axis. (3)
- (b) Write down the equation of the asymptote of the graph of $y = 2^x$. (2)
- (c) On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. (3)
- (d) Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. (2)
- (e) Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. (1)
- (f) Use Differential Calculus to verify that your answer to (e) is correct. (5)
- (Total 16 marks)**

61. The height, $h(t)$, in centimetres, of a bicycle pedal above the ground at time, t , seconds is a cosine function of the form $h(t) = A \cos(bt) + C$, where (bt) is measured in degrees.

The graph of this function for $0 \leq t \leq 4.3$ is shown below.



- (a) Write down the maximum height of the pedal above the ground. (1)
- (b) Write down the minimum height of the pedal above the ground. (1)
- (c) Find the amplitude of the function. (2)
- (d) Hence or otherwise, find the value of A and of C . (2)
- (e) (i) Write down the period of the function $h(t)$. (3)
- (ii) Hence find the value of b . (3)
- (f) Use your graphic display calculator to determine the first value of t for which the height of the pedal above the ground is 30 cm. (3)
- (g) Calculate the number of times the pedal rotates in one minute. (2)

(Total 14 marks)

62. The diagram shows triangle ABC. Point C has coordinates (4, 7) and the equation of the line AB is $x + 2y = 8$.

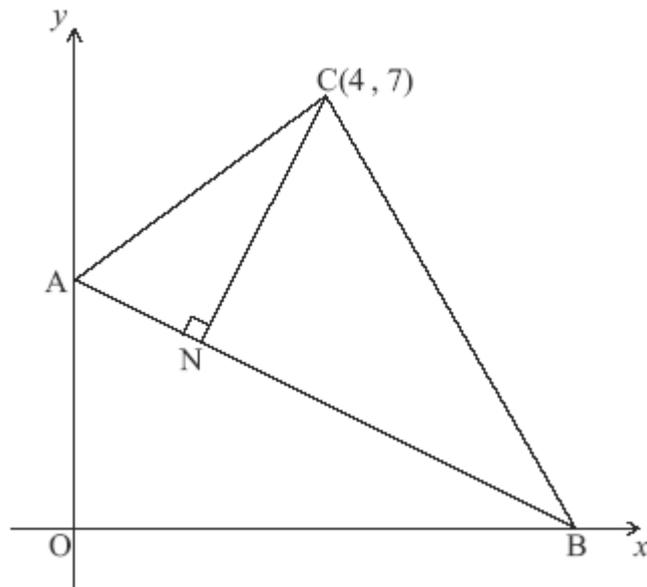


diagram not to scale

- (a) Find the coordinates of
- (i) A;
 - (ii) B.
- (2)
- (b) Show that the distance between A and B is 8.94 correct to 3 significant figures.
- (2)
- N lies on the line AB. The line CN is perpendicular to the line AB.
- (c) Find
- (i) the gradient of CN ;
 - (ii) the equation of CN.
- (5)
- (d) Calculate the coordinates of N.
- (3)
- It is known that $AC = 5$ and $BC = 8.06$.
- (e) Calculate the size of angle ACB.
- (3)
- (f) Calculate the area of triangle ACB.
- (3)

(Total 18 marks)

63. The diagram shows an office tower of total height 126 metres. It consists of a square-based pyramid VABCD on top of a cuboid ABCDPQRS.

V is directly above the centre of the base of the office tower.

The length of the sloping edge VC is 22.5 metres and the angle that VC makes with the base ABCD (angle VCA) is 53.1° .

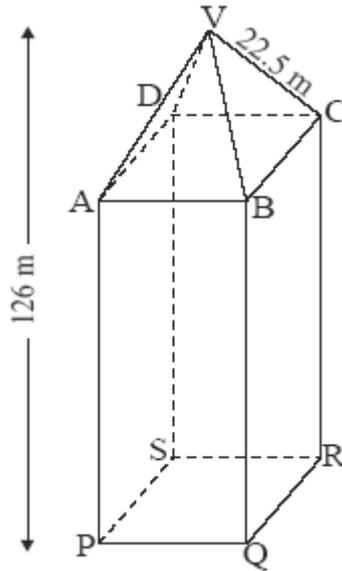


diagram not to scale

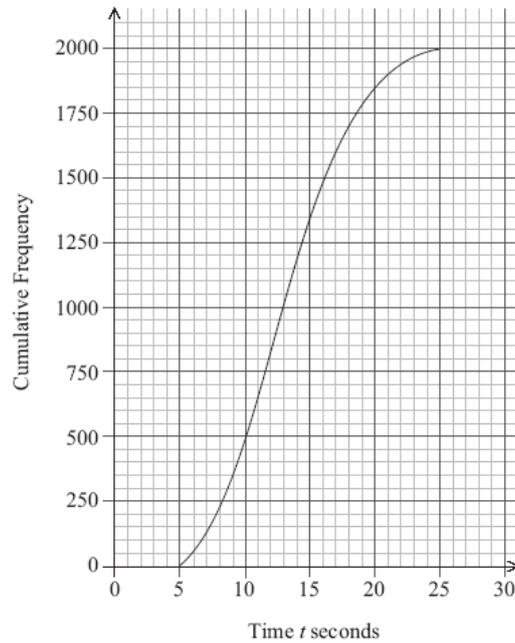
- (a) (i) Write down the length of VA in metres. (2)
- (ii) Sketch the triangle VCA showing clearly the length of VC and the size of angle VCA. (2)
- (b) Show that the height of the pyramid is 18.0 metres correct to 3 significant figures. (2)
- (c) Calculate the length of AC in metres. (3)
- (d) Show that the length of BC is 19.1 metres correct to 3 significant figures. (2)
- (e) Calculate the volume of the tower. (4)

To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90 % of the volume of the tower is occupied by air and they know that 1 m^3 of air weighs 1.2 kg.

- (f) Calculate the weight of air in the tower. (3)

(Total 16 marks)

64. The diagram shows the cumulative frequency graph for the time t taken to perform a certain task by 2000 men.



- (a) Use the diagram to estimate
- the median time;
 - the upper quartile and the lower quartile;
 - the interquartile range.
- (4)
- (b) Find the number of men who take **more than** 11 seconds to perform the task.
- (3)
- (c) 55 % of the men took less than p seconds to perform the task. Find p .
- (2)

The times taken for the 2000 men were grouped as shown in the table below.

Time	Frequency
$5 \leq t < 10$	500
$10 \leq t < 15$	850
$15 \leq t < 20$	a
$20 \leq t < 25$	b

- (d) Write down the value of
- a ;
 - b .
- (2)
- (e) Use your graphic display calculator to find an estimate of
- the mean time;
 - the standard deviation of the time.
- (3)

Everyone who performs the task in **less than** one standard deviation **below** the mean will receive a bonus. Pedro takes 9.5 seconds to perform the task.

- (f) Does Pedro receive the bonus? Justify your answer.
- (3)

(Total 17 marks)

65. The table below shows the number and weight (w) of fish delivered to a local fish market one morning.

weight (kg)	frequency	cumulative frequency
$0.50 \leq w < 0.70$	16	16
$0.70 \leq w < 0.90$	37	53
$0.90 \leq w < 1.10$	44	c
$1.10 \leq w < 1.30$	23	120
$1.30 \leq w < 1.50$	10	130

- (a) (i) Write down the value of c . (1)
- (ii) On graph paper, draw the *cumulative frequency curve* for this data. Use a scale of 1 cm to represent 0.1 kg on the horizontal axis and 1 cm to represent 10 units on the vertical axis. Label the axes clearly. (4)
- (iii) Use the graph to show that the median weight of the fish is 0.95 kg. (1)
- (b) (i) The zoo buys all fish whose weights are above the 90th percentile. How many fish does the zoo buy? (2)
- (ii) A pet food company buys all the fish in the lowest quartile. What is the maximum weight of a fish bought by the company? (3)
- (c) A restaurant buys all fish whose weights are within 10% of the median weight.
- (i) Calculate the minimum and maximum weights for the fish bought by the restaurant. (2)
- (ii) Use your graph to determine how many fish will be bought by the restaurant. (3)

(Total 16 marks)

66. In an environmental study of plant diversity around a lake, a biologist collected data about the number of different plant species (y) that were growing at different distances (x) in metres from the lake shore.

Distance (x)	2	5	8	10	13	17	23	35	40
Plant species (y)	35	34	30	29	24	19	15	13	8

- (a) Draw a scatter diagram to show the data. Use a scale of 2 cm to represent 10 metres on the x -axis and 2 cm to represent 10 plant species on the y -axis. (4)
- (b) Using your scatter diagram, describe the correlation between the number of different plant species and the distance from the lake shore. (1)
- (c) Use your graphic display calculator to write down
- \bar{x} , the mean of the distances from the lake shore;
 - \bar{y} , the mean number of plant species. (2)
- (d) Plot the point (\bar{x}, \bar{y}) on your scatter diagram. **Label this point M.** (2)
- (e) Write down the equation of the regression line y on x for the above data. (2)
- (f) Draw the regression line y on x on your scatter diagram. (2)
- (g) Estimate the number of plant species growing 30 metres from the lake shore. (2)

(Total 15 marks)

67. A university required all Science students to study one language for one year. A survey was carried out at the university amongst the 150 Science students. These students all studied one of either French, Spanish or Russian. The results of the survey are shown below.

	French	Spanish	Russian
Female	9	29	12
Male	31	40	29

Ludmila decides to use the χ^2 test at the 5 % level of significance to determine whether the choice of language is independent of gender.

- (a) State Ludmila's null hypothesis. (1)
- (b) Write down the number of degrees of freedom. (1)
- (c) Find the expected frequency for the females studying Spanish. (2)
- (d) Use your graphic display calculator to find the χ^2 test statistic for this data. (2)
- (e) State whether Ludmila accepts the null hypothesis. Give a reason for your answer. (2)

(Total 8 marks)

68. Give all your numerical answers correct to two decimal places.

On 1 January 2005, Daniel invested 30 000 AUD at an annual **simple** interest rate in a *Regular Saver* account. On 1 January 2007, Daniel had 31 650 AUD in the account.

- (a) Calculate the rate of interest. (3)

On 1 January 2005, Rebecca invested 30 000 AUD in a *Supersaver* account at a nominal annual rate of 2.5 % **compounded annually**.

- (b) Calculate the amount in the *Supersaver* account after two years. (3)
- (c) Find the number of complete years since 1 January 2005 it will take for the amount in Rebecca's account to exceed the amount in Daniel's account. (3)

On 1 January 2007, Daniel reinvested 80 % of the money from the *Regular Saver* account in an *Extra Saver* account at a nominal annual rate of 3 % **compounded quarterly**.

- (d) (i) Calculate the amount of money reinvested by Daniel on the 1 January 2007. (5)
- (ii) Find the number of complete years it will take for the amount in Daniel's *Extra Saver* account to exceed 30 000 AUD.

(Total 14 marks)