VECTORS – MAJOR IDEAS

Check answers at back of book.

1. Writing a vector $\overrightarrow{AB}$

Given point $A=(2,-3,-1)$ and $B=(4,-2,-5)$, find $\overrightarrow{AB}$. p. 332 #12a, #4

2. Find the length of $\overrightarrow{AB}$. p. 329 #2

3. Find a unit vector in the direction of $\overrightarrow{AB}$. p. 340 #8

4. Write the equation of the line through points A and B. p. 358 # 1, 3

IB Q ans. Sec. A 1.)a.) $\overrightarrow{AB} = \begin{pmatrix}-2\\3\\2\end{pmatrix}$ b.) $\vec{r} = \begin{pmatrix}3\\2\\1\end{pmatrix} + t\begin{pmatrix}-2\\3\\2\end{pmatrix}$ OR $\begin{pmatrix}1\\5\\3\end{pmatrix} + t\begin{pmatrix}-2\\3\\2\end{pmatrix}$

2.) If $\perp$, dot product $= 0$.

$\begin{pmatrix}2\\3\\-1\end{pmatrix} \cdot \begin{pmatrix}4\\1\\-p\end{pmatrix} = 8 + 3 + p = 0$

$p = -11$

b.) $q|\vec{u}| = 14$ $\quad |\vec{u}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$

$q\sqrt{14} = 14$

$q = \sqrt{14}$

5. Given that $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = -5\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, find $\mathbf{u} \cdot \mathbf{v}$. p. 344 #2
(inner product, scalar product, dot product)

6. Show that $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -5\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ are perpendicular. p. 345 #7, 9
(not enough to just show dot product is equal to zero; they need to make a connection)

7. Find p and q so that $\mathbf{u} = -5\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 10\mathbf{i} + p\mathbf{j} + q\mathbf{k}$ are parallel. p. 345 #8

8. Find the angle between $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$. (Answer: 1.33 or 75.9°)
p. 345 #11

IB Q. Sec. B.

a.) i.) $\vec{PQ} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ ii.) $\vec{r} = \begin{pmatrix} -5 \\ 11 \\ -8 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -5+t \\ 11-2t \\ -8+3t \end{pmatrix}$ ✓

b.) $-5 + t = 2 \quad t = 7$
$y_1 = 11 - 2(7) = \boxed{-3}$
$z_1 = -8 + 3(7) = \boxed{13}$

c.) set equal

d.) use $\cos\theta = \frac{|\vec{a} \cdot \vec{b}|}{|\vec{a}||\vec{b}|}$ use direction vectors as a + b

9. Find the angle between $\mathbf{r} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ -8 \end{pmatrix} + s \begin{pmatrix} -2 \\ -4 \\ 1 \end{pmatrix}$. P. 360 #2, 4

(Indicate which vectors are being used.)

10. The lines represented by $\mathbf{r} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ -5 \end{pmatrix} + s \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix}$ intersect at point T.

Find the coordinates of T. P. 370 # 1abc

TOPIC FIVE: VECTORS

SECTION A

1. The line L passes through the points A(3, 2, 1) and B(1, 5, 3).

   (a) Find the vector $\vec{AB}$.

   (b) Write down a vector equation of the line L in the form $r = a + tb$.

2 Consider the vectors $u = 2i + 3j - k$ and $v = 4i + j - pk$.

(a) Given that u is perpendicular to v find the value of p.

(b) Given that $q|u| = 14$, find the value of q.

SECTION B

[Maximum mark: 22]

3 Points P and Q have position vectors $-5i + 11j - 8k$ and $-4i + 9j - 5k$ respectively, and both lie on a line L_1.

(a) (i) Find $\vec{PQ}$.

(ii) Hence show that the equation of L_1 can be written as

$$r = (-5 + s)i + (11 - 2s)j + (-8 + 3s)k.$$

[4 marks]

The point R$(2, y_1, z_1)$ also lies on L_1.

(b) Find the value of y_1 and of z_1. *[4 marks]*

The line L_2 has equation $r = 2i + 9j + 13k + t(i + 2j + 3k)$.

(c) The lines L_1 and L_2 intersect at a point T. Find the position vector of T. *[7 marks]*

(d) Calculate the angle between the lines L_1 and L_2. *[7 marks]*